

Adaptive Bayesian Shrinkage of High-Dimensional Panel VARs

Zhiruo Rachel Zhang
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Qazi Haque

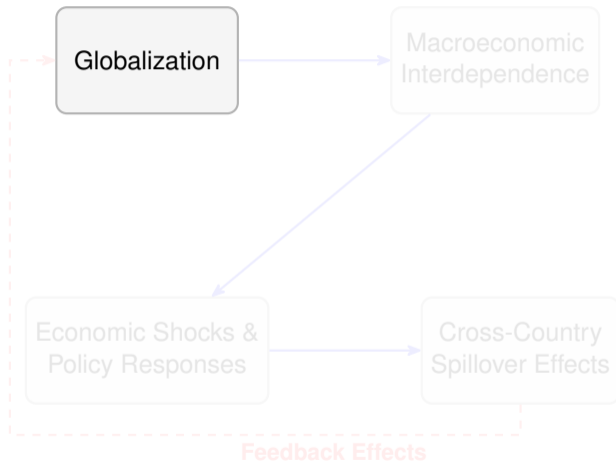
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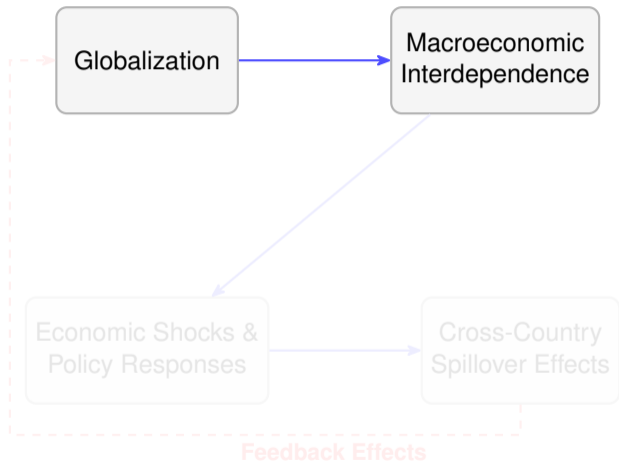
Overview



Panel Vector Autoregression(PVAR):

- **Over-Parameterization**
 - **Large parameter space:** $N \times G \times p$ parameters
 - **High dimensionality:** Many countries (N) & variables (G)
 - **Limited time series:** Short observation period (T)

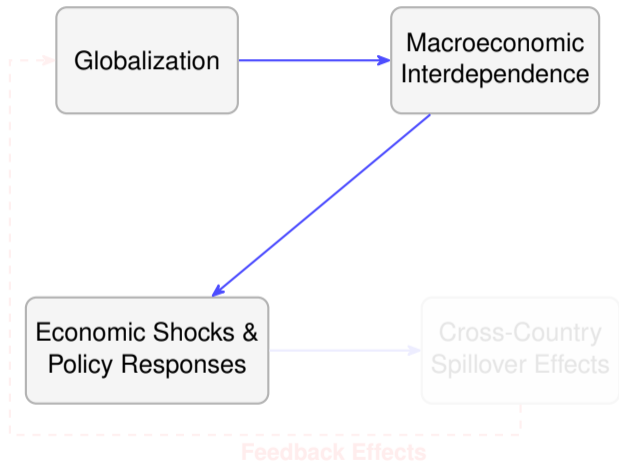
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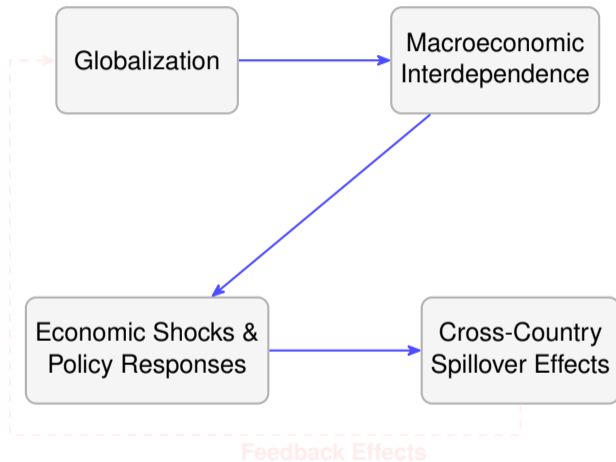
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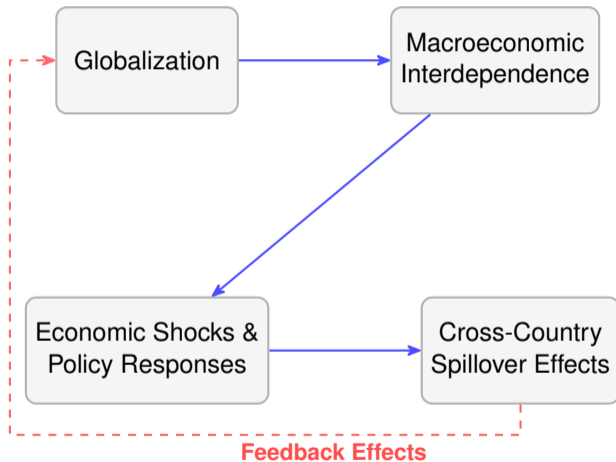
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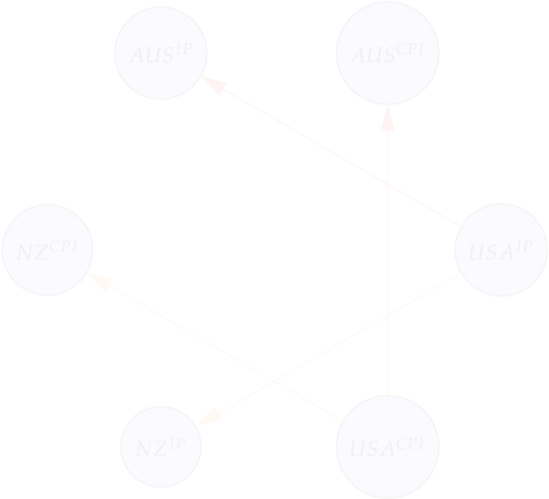
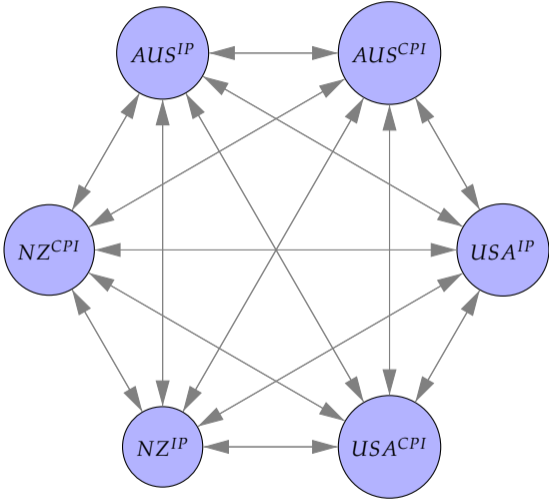
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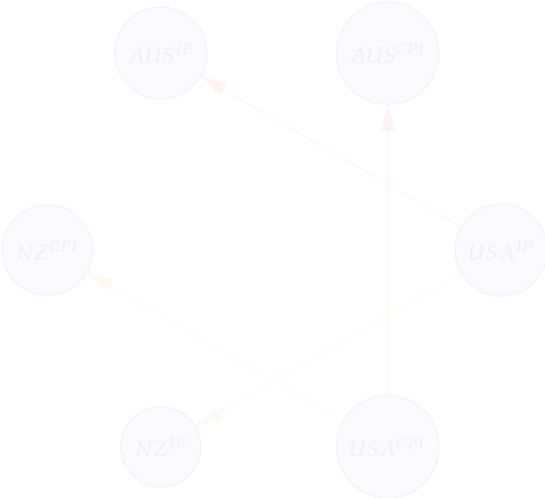
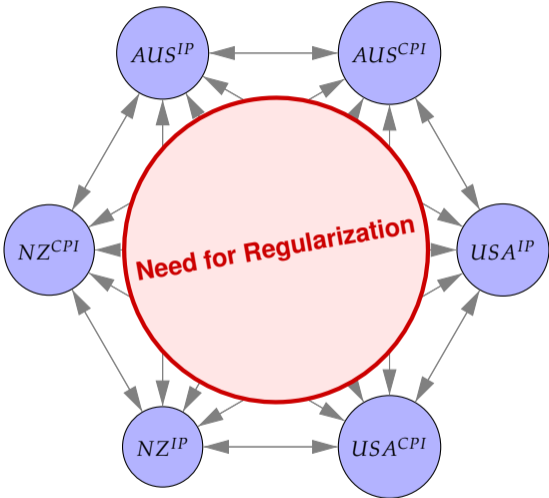
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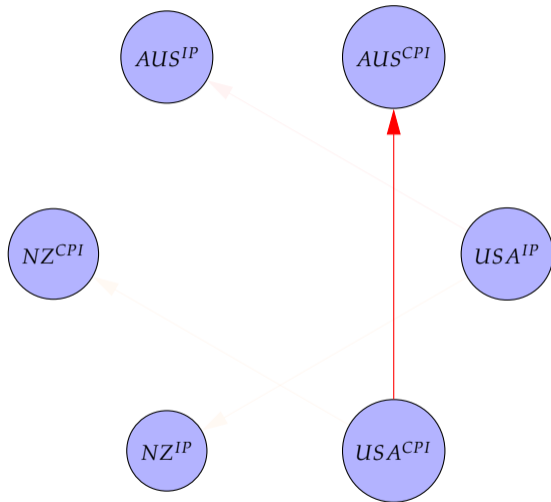
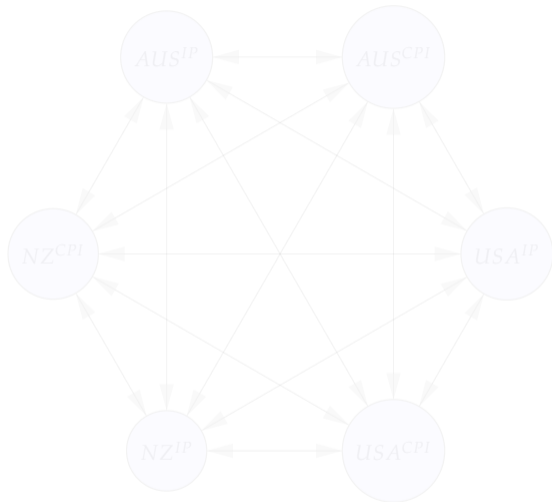
Search for Sparse Models



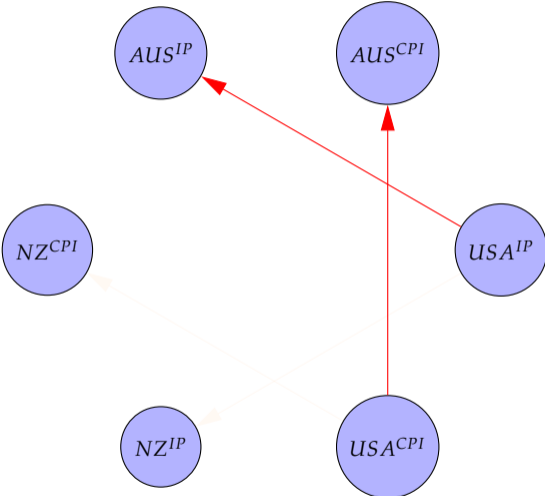
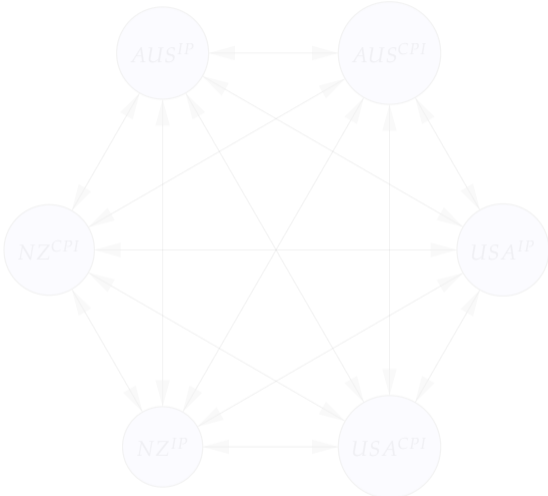
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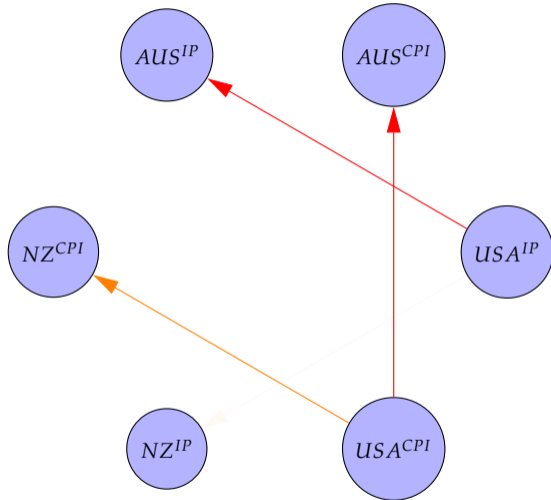
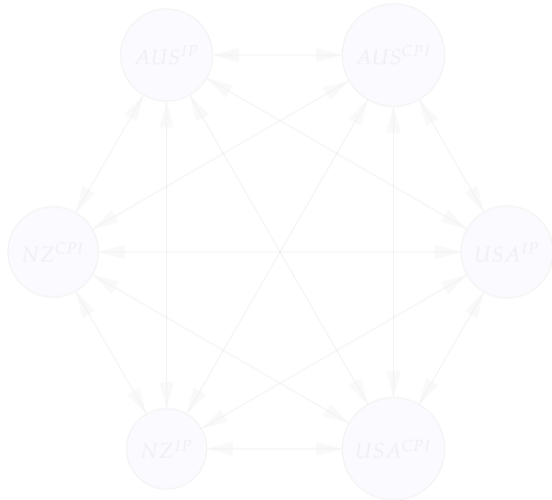
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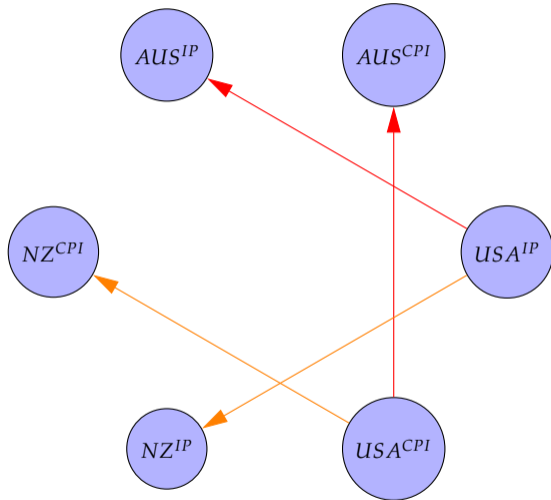
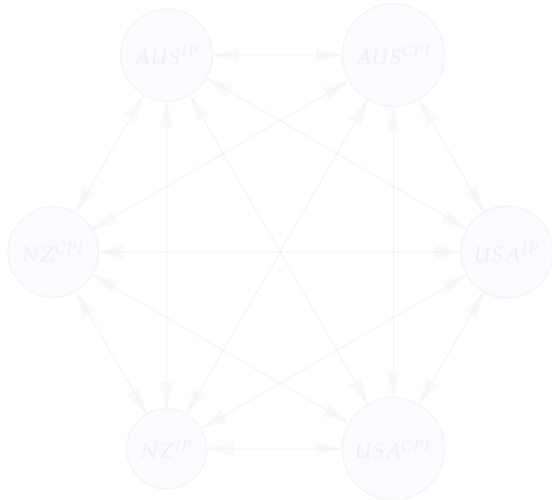
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Search for Sparse Models



Search for Sparse Models



Solving Over-Parameterization

- **The Challenge**

- **High-dimensional PVARs:** rich **spillovers** + **heterogeneity**
- **Parameter explosion:** over-fit, unstable forecasts, misleading linkages in policy & contagion analysis

- **Existing Tools**

- **Factor Models** —capture **comovement**, *no sparsity*
 - ▶ Canova and Ciccarelli, 2009, 2013
- **Bayesian Shrinkage** —**Gaussian priors**, calibrated scale
 - ▶ Koop and Korobilis, 2016
- **Lasso / Bayesian Lasso** —**sparsity**, but *ignores group/lag structure*
 - ▶ Tibshirani, 1996; Tibshirani et al., 2005; Casella et al., 2010; Ročková and George, 2018

Our Approach

Adaptive Group Shrinkage for PVARs

- **Foundation:** *Laplace spike-and-slab priors*
 - Joint Bayesian estimation: captures all dependencies in high dimensions
 - Structured sparsity: retains key spillovers
- **Mechanism:**
 - **Adaptive shrinkage:** penalty strength learned from data
 - **Group structure:** select/shrink country–lag blocks for transmission patterns
- **Implications:**
 - **Preserve heterogeneity:** retain unit-specific dynamics across countries
 - **Stable forecasts:** reliable in high-dimensional,
 - **Interpretable linkages:** map contagion and policy transmission channels

Model & Methodology

Model Equation for country i :

$$y_{it} = A_{1,i} Y_{t-1} + \cdots + A_{p,i} Y_{t-p} + \epsilon_{it}, \quad (1)$$

$[G \times 1] \quad [G \times NG][N \times 1] \quad [G \times NG][N \times 1] \quad [G \times 1]$

where $Y_t = (y'_{1t}, \dots, y'_{Nt})'$.

Pooling countries together...

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + \epsilon_t, \quad \epsilon_t \sim (0, \Sigma). \quad (2)$$

Example

Consider $N = 3, G = 2, p = 1$

$$\begin{bmatrix} GDP_t^{AUS} \\ CPI_t^{AUS} \\ GDP_t^{NZ} \\ CPI_t^{NZ} \\ GDP_t^{US} \\ CPI_t^{US} \end{bmatrix} = \begin{bmatrix} \alpha_{11}^{11} & \alpha_{12}^{11} & \alpha_{11}^{12} & \alpha_{12}^{12} & \alpha_{11}^{13} & \alpha_{12}^{13} \\ \alpha_{21}^{11} & \alpha_{22}^{11} & \alpha_{21}^{12} & \alpha_{22}^{12} & \alpha_{21}^{13} & \alpha_{22}^{13} \\ \alpha_{11}^{21} & \alpha_{12}^{21} & \alpha_{11}^{22} & \alpha_{12}^{22} & \alpha_{11}^{23} & \alpha_{12}^{23} \\ \alpha_{21}^{21} & \alpha_{22}^{21} & \alpha_{21}^{22} & \alpha_{22}^{22} & \alpha_{21}^{23} & \alpha_{22}^{23} \\ \alpha_{11}^{31} & \alpha_{12}^{31} & \alpha_{11}^{32} & \alpha_{12}^{32} & \alpha_{11}^{33} & \alpha_{12}^{33} \\ \alpha_{21}^{31} & \alpha_{22}^{31} & \alpha_{21}^{32} & \alpha_{22}^{32} & \alpha_{21}^{33} & \alpha_{22}^{33} \end{bmatrix} \begin{bmatrix} GDP_{t-1}^{AUS} \\ CPI_{t-1}^{AUS} \\ GDP_{t-1}^{NZ} \\ CPI_{t-1}^{NZ} \\ GDP_{t-1}^{US} \\ CPI_{t-1}^{US} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t}^{AUS} \\ \epsilon_{2t}^{AUS} \\ \epsilon_{1t}^{NZ} \\ \epsilon_{2t}^{NZ} \\ \epsilon_{1t}^{US} \\ \epsilon_{2t}^{US} \end{bmatrix}$$

- α_{lk}^{ij} : coefficient of variable k of country j appears on the equation of variable l of country i .

Type of Restrictions

Canova and Ciccarelli (2013); Koop and Korobilis (2016)

- **Dynamic Interdependencies (DI):** Links across countries through PVAR coefficients.
 - Number of possible DIs: $N(N - 1)$
 - **Example:** Will changes in the Consumer Price Index in the U.S. spill over to New Zealand in the subsequent month?

▶ [Click to DI Example](#)

- **Cross-national Homogeneity (CSH):** VAR coefficients differ across countries.
 - Number of possible CSHs: $\frac{N(N-1)}{2}$
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Example: DI Restriction

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Methodological Contributions

Panel Spike-and-Slab Lasso (PSSL) with Structured Shrinkage

- **DI Shrinkage**

$$\mathbf{A}_{ij} \sim (1 - \gamma^{DI}) \text{Laplace}^{\text{Spike}} + \gamma^{DI} \text{Laplace}^{\text{Slab}}$$

- **CSH Shrinkage**

- **Group Average Shrinkage (GAS)**

$$\mathbf{A}_{ii} - \bar{\mathbf{A}} \sim (1 - \gamma^{\text{GAS}}) \text{Laplace}^{\text{Spike}} + \gamma^{\text{GAS}} \text{Laplace}^{\text{Slab}}$$

- **Group Value Shrinkage (GVS)**

$$\mathbf{A}_{ii} - \mathbf{A}_{jj} \sim (1 - \gamma^{\text{GVS}}) \text{Laplace}^{\text{Spike}} + \gamma^{\text{GVS}} \text{Laplace}^{\text{Slab}}$$

$\text{Laplace}^{\text{Spike}}$ = strong shrinkage, $\text{Laplace}^{\text{Slab}}$ = weak shrinkage, γ = selection indicator.

Penalty Function

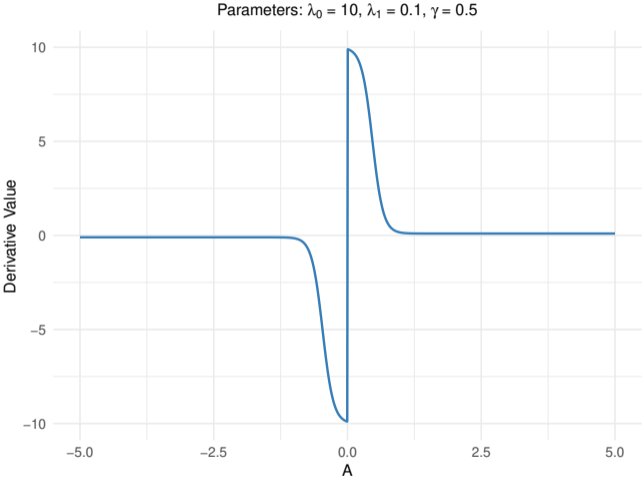


Figure: Derivative of $pen_{DI}(A)$

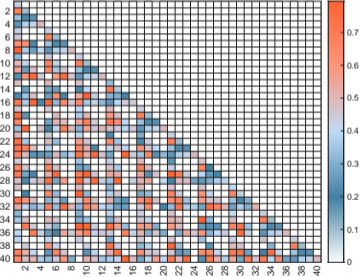
Monte Carlo Experiment

Simulations

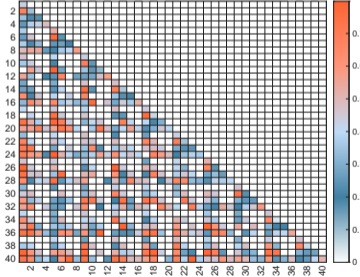
- **Model:** PVAR(1).

$$Y_t = AY_{t-1} + \epsilon_t$$

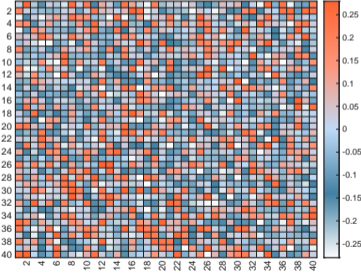
- Four settings: $(N, G, T) = (5, 2, 100), (10, 2, 100), (5, 4, 100), (10, 4, 100)$.
- DGP of **A**:



DI and CSH

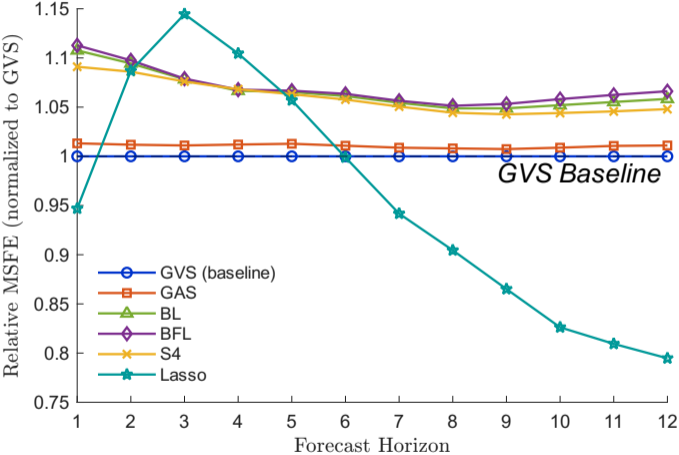


DI



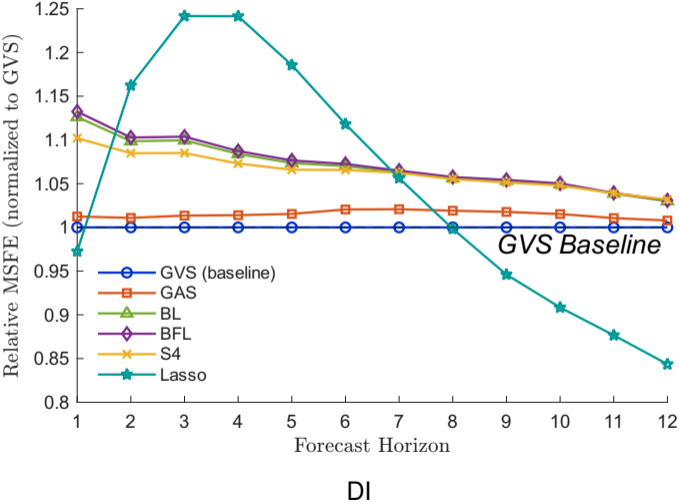
CSH

Relative RMSFE: Group Value Shrinkage (GVS)

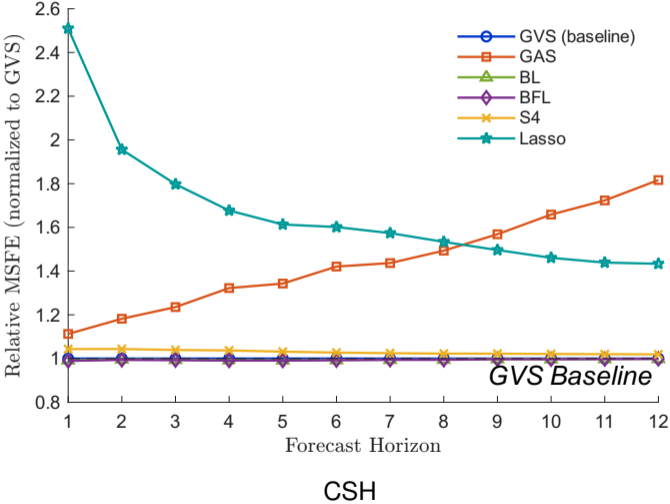


DI and CSH

Relative RMSFE: Group Value Shrinkage (GVS)



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GVS Baseline

Eurozone Financial Contagion

Key Economic Indicators

- **Three Variables:**

- 10-year sovereign bond yield.
- Growth rate of industrial production.
- Average bid-ask spread across sovereign bonds.

- **Ten Countries:**

- **Core Countries:** Austria (AUT), Belgium (BEL), Finland (FIN), France (FRA), Netherlands (NLD).
- **Peripheral Countries:** Greece (GRC), Ireland (IRE), Italy (ITA), Portugal (PRT), Spain (ESP).

- **Time Period:**

- **2001M1 - 2019M12**

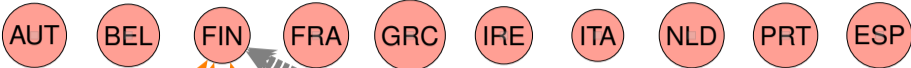
Model Selection: Marginal Likelihood Ranking

Table: Ranking of PVAR Specifications (Gelfand–Dey Method)

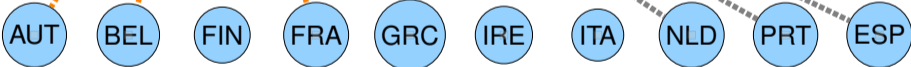
Rank	Model & Restrictions	Marginal Log-Likelihood
1	DI only (PSSL)	-38.46
2	GVS	-41.01
3	S4–Normal prior	-42.41
4	Unrestricted PVAR	-43.65
5	CSH only (GVS)	-49.48
6	GAS	-50.64
7	CSH only (GAS)	-50.76

Countries with DI restrictions

Present



Past



→ Strong DI: 95% and above

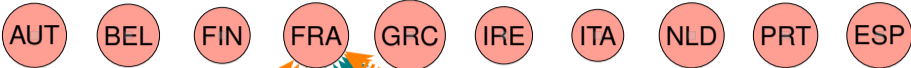
---→ Moderate DI: 75% to 94%

.....→ Weak DI: 50% to 74%

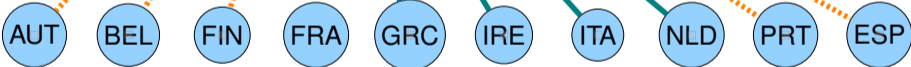
○ No statistically DI: Below 50% (no link)

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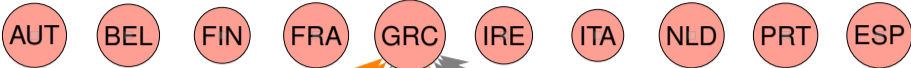
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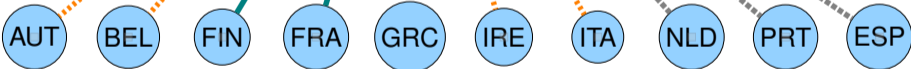
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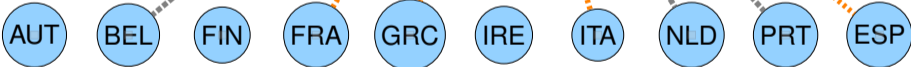
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- **Innovation: PVAR + Adaptive Group shrinkage methods**
 - **Efficiency:** Improved estimation and forecast performance.
 - **Flexibility:** Adaptable to various sparsity structures.
- **Financial Contagion in Euro Area Bonds**
 - **Persistent core** → **periphery contagion** is the dominant pattern
 - **Periphery linkages are weaker**; reverse contagion is limited and crisis-driven
 - **Network is asymmetric and hierarchical** –shocks originate in the core and propagate outward

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Thank You!

Questions or Feedback?

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References I

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